MATHEMATICS

Mob. : 9470844028
9546359990

AIM POINTS
MATHEMATICS
DIR. FIROZ AHMAD

Ram Rajya More, Siwan

XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XII (PQRS)

DEFINITE INTEGRALS

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. Let $\phi(x)$ be the primitive or antiderivative of a function f(x) defined on [a, b] i.e., $\frac{d}{dx}\{\phi(x)\} = f(x)$.

   Then the definite integral of f(x) over [a, b] is denoted by $\int_a^b f(x)\,dx$ and is equal to $[\phi(b) - \phi(a)]$.

   i.e., $\int_a^b f(x)\,dx = \phi(b) - \phi(a)$

2. Following are some fundamental properties of definite integrals which are very useful inevaluating integrals.

   (i) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ i.e., integration is independent of the change of variable.

   (ii) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

   i.e., if the limits of a definite integral are interchanged then its value changes by minus sign only.

   (iii) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where a > c > b.

   The above property can be generalized intothe following form

   $$\int_a^b f(x)\,dx = \int_a^{c_1} f(x)\,dx + \int_{c_1}^{c_2} f(x)\,dx + \ldots + \int_{c_n}^{b} f(x)\,dx$$

   where $a < c_1 < c_2 < c_3 \ldots < c_{n-1} < c_n < b$.

   (iv) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

   (v) $$\int_{-a}^a f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is an even function} \\ 0 & \text{, if } f(x) \text{ is an odd function} \end{cases}$$

   (vi) $$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(2a-x) = f(x) \\ 0 & \text{, if } f(2a-x) = -f(x) \end{cases}$$

   (vii) $\int_0^{2a} f(x)\,dx = \int_0^{2a} \{f(x) + f(2a-x)\}\,dx$

   (viii) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(ix) $\int_a^b f(x)\,dx = (b-a)\int_0^1 f\{(b-a)x+a\}\,dx$

3. If f(x) is a real valued continuous function defined on [a, b] which is divided into n equal parts each of width h by inserting (n–1) points a + h, a + 2h ...., a + (n–1) h between a and b. Then,

$\int_a^b f(x)\,dx = \lim_{h\to 0}$ [f(a) + f(a + h) + (f(a + 2h) + ... + f[(a + (n – h)], where $h = \frac{b-a}{h}$.

EXERCISE-1

1. Let θ (x) be the primitive or antiderivative of a continous function f(x) defined on [a, b] i.e., $\frac{d}{dx}\{\phi(x)\} = f(x)$. Then the definite integral of f(x) over [a, b] is denoted by $\int_a^b f(x)\,dx$ and is equal to [ϕ(b) - ϕ(a)].

2. To evaluate the definite integral $\int_a^b f(x)\,dx$ of a continuous function f(x) definedon [a, b] we use the following algorithm.

3. Evaluate :

(i) $\int_1^2 x^2 dx$ (ii) $\int_{-4}^{-1} \frac{1}{x}dx$

4. Evaluate :

(i) $\int_0^{\pi/4} \tan^2 x\,dx$ (ii) $\int_0^{\pi/4} \sin 3x \sin 2x\,dx$

5. Evaluate : $\int_0^{\pi/2} \cos^3 x\,dx$

6. Evaluate : $\int_0^{\pi/4} \sqrt{1-\sin 2x\,dx}$

7. Evaluate : $\int_0^{\pi/2} \sqrt{1-\cos 2x}\,dx$

8. If $\int_a^b x^3\,dx = 0$ and if $\int_a^b x^2\,dx = \frac{2}{3}$, find a and b.

9. If $\int_0^a \sqrt{x}\,dx = 2a\int_0^{\pi/2} \sin^3 x\,dx$, find the value of $\int_a^{a+1} x\,dx$.

10. Evaluate : $\int_0^4 \frac{1}{\sqrt{x^2+2x+3}}dx$

11. Evaluate :

(i) $\int_{1/4}^{1/2} \frac{dx}{\sqrt{x-x^2}} dx$ (ii) $\int_0^2 \frac{5x+1}{x^2+4} dx$

12. Evaluate :

(i) $\int_0^1 x\, e^x\, dx$ (ii) $\int_0^1 \left\{xe^x + \sin\frac{\pi x}{4}\right\} dx$

13. Evaluate :

(i) $\int_1^2 \frac{5x^2}{x^2+4x+3} dx$ (ii) $\int_1^3 \frac{1}{x^2(x+1)} dx$

14. Evaluate : $\int_0^{\pi/6} (2+3x^2)\cos 3x\, dx$

15. Evaluate : $\int_0^{\infty} \frac{1}{(x^2+a^2)(x^2+b^2)} dx$.

16. If (x) is of the form $f(x) = a + bx + cx^2$, show that

$$\int_0^1 f(x)\, dx = \frac{1}{6}\left[f(0) + 4f\left(\frac{1}{2}\right) + f(1)\right]$$

17. Evaluate : $\int_1^2 \frac{1}{x(1+x^2)} dx$

18. Evaluate : $\int_{\pi/4}^{\pi/2} \cos 2x \log \sin x\, dx$

19. $\int_{\pi/6}^{\pi/4} \operatorname{cosec} x\, dx$

20. $\int_0^{\pi/2} \cos^2 x\, dx$

21. $\int_0^{\pi/2} a^2\cos^2 x + b^2\sin^2 x)dx$

22. $\int_0^{\pi/2} \sqrt{1+\sin x}\, dx$

23. $\int_e^{e^2} \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2} dx\right\}$

24. $\int_0^1 \frac{2x+3}{5x^2+1} dx$

25. $\int_0^2 \frac{1}{4+x-x^2}dx$

25. $\int_0^2 \frac{1}{\sqrt{3+2x-x^2}}dx$

26. If $\int_0^k \frac{1}{2+8x^2}dx = \frac{\pi}{16}$, find the value of k.

27. $\int_1^2 \left(\frac{x-1}{x^2}\right)e^x dx$

28. $\int_0^1 \left(xe^{2x} + \sin\frac{\pi x}{2}\right)dx$

29. $\int_{\pi/2}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x}\right)dx$

30. $\int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$

31. If $\int_0^a 3x^2 dx = 8$, find the value of a.

32. $\int_1^2 \frac{x}{(x+1)(x+2)}dx$

33. $\int_0^{\pi/2} \sin^3 x\, dx$

34. $\int_0^{\pi} \left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right)dx$

35. $\int_1^2 e^{2x}\left(\frac{1}{x} - \frac{1}{2x^2}\right)dx$

36. $\int_{-1}^1 \frac{1}{x^2+2x+5}dx$

37. $\int_0^1 \frac{1}{\sqrt{1+x}-\sqrt{x}}dx$

38. When the variable in a definite integral is changed, the substitution in terms of new variable should be effected at three places.
   (i) in the integrand,

(ii) in the differential, say, dx

(iii) in the limits

The limits of the new variable, say, t are simply the values of t corresponding to the values of the original variable, say, x, and so they can be easily obtained by putting the values of x in the substitutional relation between x and t. The method is illustrated in the following examples.

39. Evaluate : $\int_0^{\pi/2} \frac{\cos\theta}{(1+\sin\theta)(2+\sin\theta)} d\theta$

40. Evaluate :

(i) $\int_0^a \frac{x^4}{\sqrt{a^2-x^2}} dx$ (ii) $\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}} dx$ (iii) $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

41. Evaluate :

(i) $\int_0^{\pi/4} \tan^3 x\, dx$ (ii) $\int_0^{\pi/2} \left(\sqrt{\tan x}+\sqrt{\cot x}\right) dx$

42. Evaluate : $\int_0^{\pi} \frac{1}{5+4\cos x} dx$

43. Evaluate :

(i) $\int_0^{\pi/2} \frac{1}{2\cos x+4\sin x} dx$ (ii) $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$

44. Evaluate :

(i) $\int_0^{\pi/2} \frac{\cos x}{(3\cos x+\sin x)} dx$ (ii) $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x} dx$

45. Evaluate : $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x+\cos^4 x} dx$

46. Evaluate : $\int_0^1 x\frac{\sqrt{1-x^2}}{1+x^2} dx$

47. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{\sin x+\cos x}{\sqrt{\sin 2x}} dx$

48. Evaluate : $\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x+4\sin^2 x} dx$

49. If $I_n = \int_0^{\pi/4} \tan^n x\, dx$, show that $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}, \frac{1}{I_5+I_7}, \ldots\ldots\ldots$ from an A.P. Find the common difference of this progression.

50. $\int_2^4 \frac{x}{x^2+1} dx$

51. $\int_0^{\pi/2} \frac{1}{5\cos x + 3\sin x} dx$

52. $\int_0^1 \frac{e^x}{1+e^{2x}} dx$

53. $\int_0^1 xe^{x^2} dx$

54. $\int_0^{\pi/2} \sqrt{\sin\phi} \cos^5 \phi \, d\phi$

55. $\int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx$

56. $\int_0^2 x\sqrt{x+2} dx$

57. $\int_0^{\pi/2} \frac{\sin x \cos x}{1+\sin^4 x} dx$

58. $\int_0^{\pi/2} \frac{dx}{a\cos x + b\sin x}.a, b > 0$

59. $\int_0^{\pi} \frac{\sin x}{\sin x + \cos x} dx$

60. $\int_0^{\pi} \frac{1}{3+2\sin x + \cos x} dx$

61. $\int_0^1 \tan^{-1} x \, dx$

62. $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x) dx$

63. $\int_0^{\pi} \frac{1}{5+3\cos x} dx$

64. $\int_0^{\pi/2} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$

65. $\int_0^{\pi} \sin^3 x(1+2\cos x)(1+\cos x)^2 dx$

66. $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{3/2}} dx$

67. $\int_0^1 x \tan^{-1} x \, dx$

68. $\int_0^{\pi/2} x^2 \sin x \, dx$

69. $\int_{-a}^{a} \frac{\sqrt{1-x}}{1+x} dx$

70. $\int_0^{\pi} 5(5-4\cos\theta)^{1/4} \sin\theta \, d\theta$

71. $\int_1^2 \frac{1}{x(1+\log x)^2} dx$

72. $\int_4^9 \frac{\sqrt{x}}{(30-x^{3/2})^2} dx$

73. $\int_0^{\pi/4} \sin^3 2t \cos 2t \, dt$

74. $\int_{-1}^{1} 5x^4 \sqrt{x^5+1} \, dx$

75. $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

76. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ i.e., integration is independent of the change of variable.

77. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where $a < c < b$.

78. Evaluate :

(i) $\int_{-1}^{1} f(x)\,dx$, where $f(x) = \begin{cases} 1-2x, & x \le 0 \\ 1+2x, & x \ge 0 \end{cases}$

(ii) $\int_1^4 f(x)\,dx$, where $f(x) = \begin{cases} 2x+8, & 1 \le x \le 2 \\ 6x \quad, & 2 \le x \le 4 \end{cases}$

79. Evaluate :

(i) $\int_0^{\pi} |\cos x|\, dx$

(ii) $\int_0^{2} |x^2+2x-3|\, dx$

(iii) $\int_1^{4} (|x-1|+|x-2|+|x-3|)\, dx$

(iv) $\int_{-1}^{2} |x^3-x|\, dx$

80. If [-] denotes the greates integer function, then find the value of $\int_1^2 [3x]dx$.

81. Let f(x) = x − [x], for every real number x, where [x] is the integral part of x. Then, evaluate $\int_{-1}^{1} f(x)\, dx$.

82. Evaluate :

(i) $\int_0^{\sqrt{3}} \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$

(ii) $\int_0^{1} \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$

83. Prove that $\int_{1/e}^{\tan x} \frac{t}{1+t^2} dt + \int_{1/e}^{\cot x} \frac{1}{t(1+t^2)} dt = 1$ for all x for which tan x and cot x are defined.

84. If f(x) is a continuous function defined on [a, b], then $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

85. Evaluate :

(i) $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} dx$

(ii) $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}} dx$

86. Prove that $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} dx = \frac{b-a}{2}$

87. Prove that $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx = \frac{\pi}{4}$

88. Evaluate : $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

89. Evaluate :

(i) $\int_0^{\pi/2} \log \tan x\, dx$

(ii) $\int_0^{\pi/4} \log(1+\tan x)\, dx$

90. Evaluate :

(i) $\int_0^{\pi/2} \frac{\sin x-\cos x}{1+\sin x\cos x} dx$

(ii) $\int_0^{\pi/2} (2\log \sin x - \log \sin 2x)dx$

91. Prove that : $\int_0^{2a} f(x)dx = \int_0^{2a} f(2a-x)dx.$

92. Evaluate : $\int_0^1 x(1-x)^n dx$

93. Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$

94. Evaluate :

(i) $\int_0^{\pi/2} \sin 2x \log \tan x \, dx = 0$

(ii) $\int_0^1 \log\left(\frac{1}{x}-1\right) dx = 0$

95. Evaluate :

(i) $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$

(ii) $\int_0^{\pi/2} \frac{\sin^2 x}{1+\sin x \cos x} dx$

96. Evaluate : $\int_0^1 \cot^{-1}(1-x+x^2) dx$

97. Let $I = \int_0^a x f(x) dx$ where f(x) is a function of x whose integral is known and f(a – x) = f(x).

98. Evaluate :

(i) $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x} dx$

(ii) $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

99. Evaluate :

(i) $\int_0^{\pi} \frac{x}{1+\sin x} dx$

(ii) $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

100. If f and g are continuous on [0, a] and satisfy f(x) = f(a – x) and g(x) + g(a – x) = 2, show that

$$\int_0^a f(x)g(x)dx = \int_0^a f(x)dx$$

101. Evaluate : $\int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$

102. Prove that : $\int_0^{\pi} \frac{x}{1-\cos\alpha \sin x} dx = \frac{\pi(\pi-\alpha)}{\sin\alpha}$

103. Evaluate :

(i) $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$

(ii) $\int_{-\pi/2}^{\pi/2} \sin^2 x \, dx$

104. Evaluate :

(i) $\int_{-\pi/4}^{\pi/4} x^3 \sin^4 x\,dx$ (ii) $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\,dx$

105. Evaluate : $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$

106. Evaluate : $\int_{-1}^{3/2} |x \sin \pi x|\,dx$

107. If f(x) is a continuous function defined on [0, 2a], then

$$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$$

108. Evaluate : $\int_0^{2\pi} \cos^5 x\,dx.$

109. Evaluate : $\int_0^{\pi} \log(1+\cos x)dx$

110. For x > 0, let f(x) = $\int_1^x \frac{\log_e t}{1+t}dt$. Find the function f(x) + $f\left(\frac{1}{x}\right)$ and show that $f(e)+f\left(\frac{1}{e}\right) = \frac{1}{2}$.

111. Show that : $\int_0^{\pi/2} f(\sin 2x)\sin x\,dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x\,dx$

112. $\int_1^4 f(x)$, where $f(x) = \begin{cases} 4x+3, \text{if } 1 \le x \le 2 \\ 3x+5, \text{if } 2 \le x \le 4 \end{cases}$

113. $\int_{-3}^{3} |x+1|\,dx$

114. $\int_{-1}^{1} |2x+1|\,dx$

115. $\int_0^2 |x^2-3x+2|\,dx$

116. $\int_{-2}^{2} |x+1|\,dx$

117. $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\,dx$

118. $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx$

119. $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

120. $\int_0^\infty \frac{x}{(1+x)(1+x^2)} dx$

121. $\int_0^\pi \frac{x \tan x}{\sec x \operatorname{cosec} x} dx$

122. $\int_0^\pi \frac{x \sin x}{1+\sin x} dx$

123. $\int_0^\pi \frac{x}{1+\cos\alpha \sin x} dx, \; 0 < \alpha < \pi$

124. $\int_0^{\pi/2} (2\log\cos x - \log\sin 2x) dx$

125. $\int_{-1}^1 \log\left(\frac{2-x}{2+x}\right) dx$

126. $\int_0^\pi \log(1-\cos x) dx$

127. $\int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin x}{2+\sin x}\right) dx$

128. $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$

129. $\int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}} dx$

130. $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}} dx$

131. $\int_{-\pi/2}^{\pi/2} \{\sin|x| + \cos|x|\} dx$

132. $\int_0^2 x\sqrt{2-x}\, dx$

133. $\int_{-5}^{0} f(x)dx$, where $f(x) = |x| + |x+2| + |x+5|$

134. $\int_{0}^{4} |x-1|dx$

135. If f is an integrable function such that f(2a – x) = f(x), then prove that $\int_{0}^{2a} f(x)dx = 2\int_{0}^{a} f(x)dx$

136. If f is an integrable function, show that

(i) $\int_{-a}^{a} f(x^2)dx = 2\int_{0}^{a} f(x^2)dx$

(ii) $\int_{-a}^{a} x f(x^2)dx = 0$

137. If f(x) is a continuous function defined on [–a, a], then prove that $\int_{-a}^{a} f(x)dx = \int_{0}^{a} \{f(x)+f(-x)\}dx$

138. Evaluate the following integrals as limit of sums :

(i) $\int_{0}^{2} (x+4)dx$

(ii) $\int_{0}^{2} (2x+1)dx$

139. Evaluate the following integrals as limit of sums :

(i) $\int_{0}^{2} (x^2+3)dx$

(ii) $\int_{1}^{3} (2x^2+5)dx$

140. Evaluate the following integrals as limit of sums :

(i) $\int_{1}^{3} (x^2+x)dx$

(ii) $\int_{1}^{3} (x^2+5x)dx$

141. Evaluate the following integrals as limit of sums : $\int_{-1}^{1} e^x dx$

142. Evaluate : $\int_{a}^{b} \sin x\, dx$ as limit of sums.

143. Evaluate the following integrals as limit of sums : $\int_{0}^{1} e^{2-3x} dx$

144. $\int_{0}^{3} (x+4)dx$

145. $\int_{1}^{3} (3x-2)dx$

146. $\int_{1}^{4} (x^2-x)dx$

147. $\int_0^2 e^x dx$

148. $\int_0^2 (3x^2 - 2)dx$

149. $\int_0^2 (x^2 + 2)dx$

150. $\int_0^4 (x + e^{2x})dx$

151. $\int_0^2 (x^2 + x)dx$

152. $\int_0^2 (x^2 + 2x + 1)dx$

153. $\int_0^3 (2x^2 + 3x + 5)dx$

154. $\int_a^b x\,dx$

155. $\int_0^5 (x + 1)dx$

156. $\int_2^3 x^2 dx$

157. $\int_1^4 (x^2 - x)dx$

158. Write the value of $\int_{-\pi/2}^{\pi/2} \sin^2 x\,dx$

159. Write the value of $\int_{-\pi/2}^{\pi/2} \cos^2 x\,dx$

160. Write the value of $\int_0^4 \frac{1}{\sqrt{16 - x^2}} dx$

161. Write the value of $\int_0^{\pi/2} \sqrt{1 - \cos 2x}\,dx$

162. Write the value of $\int_0^{\pi/2} \log\left(\frac{3+5\cos x}{3+5\sin x}\right)dx$

163. Write the value of $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x+\cos^n x}dx, n \in N.$

164. Write the value of $\int_0^{\pi} \cos^5 x\,dx$

165. Write the value of $\int_0^{2} [x]dx$

166. Write the value of $\int_{-\pi/2}^{\pi/2} \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)d\theta$

EXERCISE-2

1. The value of $\int_0^{2\pi} \sqrt{1+\sin\frac{x}{2}}dx$ is

(a) 0 (b) 2 (c) 8 (d) 4

2. The value of the integral $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ is

(a) 0 (b) $\pi/2$ (c) $\pi/4$ (d) none of these

3. $\int_0^{\infty} \frac{1}{1+e^x}dx$ equals

(a) 2 (b) 1 (c) $\pi/4$ (d) $\pi^2/8$

4. $\int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$ equals

(a) 2 (b) 1 (c) $\pi/4$ (d) $\pi^2/8$

5. $\int_0^{\pi/2} \frac{1}{2+\cos x}dx$ equals

(a) $\log\left(\frac{2}{3}\right)$ (b) $\log\left(\frac{3}{2}\right)$ (c) $\log\left(\frac{3}{4}\right)$ (d) $\left(\frac{4}{3}\right)$

6. Given that $\int_0^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)(x^2+c^2)}dx = \frac{\pi}{2(a+b)(b+c)(c+a)}$ the value of $\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$, is

(a) $\frac{\pi}{60}$ (b) $\frac{\pi}{20}$ (c) $\frac{\pi}{40}$ (d) $\frac{\pi}{80}$

7. $\int_0^3 \frac{3x+1}{x^2+9}dx =$

(a) $\frac{\pi}{12}+\log\left(2\sqrt{2}\right)$ (b) $\frac{\pi}{2}+\log\left(2\sqrt{2}\right)$ (c) $\frac{\pi}{6}+\log\left(2\sqrt{2}\right)$ (d) $\frac{\pi}{3}+\log\left(2\sqrt{2}\right)$

8. The value of the integral $\int_0^\infty \frac{x}{(1+x)(1+x^2)}dx$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) none of these

9. $\int_{-\pi/2}^{\pi/2} \sin|x|dx$ is equal to

(a) 1 (b) 2 (c) –1 (d) –2

10. $\int_0^{\pi/2} \frac{1}{1+\tan x}dx$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

11. The falue of $\int_0^{\pi/2} \cos x\ e^{\sin x}\, dx$ is

(a) 1 (b) e – 1 (c) 0 (d) –1

12. If $\int_0^a \frac{1}{2+4x^2}dx = \frac{\pi}{8}$, then a euqals

(a) $\frac{\pi}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\pi}{4}$ (d) 1

13. If $\int_0^1 f(x)dx = 1, \int_0^1 xf(x)dx = a, \int_0^1 x^2f(x)dx = a^2$, then $\int_0^1 (a-x)^2 f(x)dx$ equals

(a) $4a^2$ (b) 0 (c) $2a^2$ (d) none of these

14. The value of $\int_{-\pi}^{\pi} \sin^3 x \cos^2 x\, dx$ is

(a) $\frac{\pi^4}{2}$ (b) $\frac{\pi^4}{4}$ (c) 0 (d) none of these

15. $\int_{\pi/6}^{\pi/3} \frac{1}{\sin 2x}dx$ is equal to

(a) $\log_e 3$ (b) $\log_e \sqrt{3}$ (c) $\frac{1}{2}\log(-1)$ (d) $\log(-1)$

16. $\int_0^{\pi/2} \frac{1}{1+\cot^3 x} dx$ is equal to

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

17. $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ is equal to

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

18. $\int_0^1 \frac{d}{dx}\left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\} dx$ is equal to

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

19. The value of $\int_0^1 \frac{1}{5+3\cos x} dx$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{2}$ (d) 0

20. $\int_0^\infty \log\left(x+\frac{1}{x}\right)\frac{1}{1+x^2} dx =$

(a) $\pi \ln 2$ (b) $-\pi \ln 2$ (c) 0 (d) $-\frac{\pi}{2}\ln 2$

21. If $(a + b - x) = f(x)$, then $\int_a^b x f(x)\, dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (b) $\frac{a+b}{2}\int_a^b f(b+x)dx$ (c) $\frac{b-a}{2}\int_a^b f(x)dx$ (d) $\frac{a+b}{2}\int_a^b f(x)dx$

22. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$, is

(a) 1 (b) 0 (c) -1 (d) $\frac{\pi}{4}$

23. The value of $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ is

(a) 2 (b) $\frac{3}{4}$ (c) 0 (d) -2

EXERCISE-3

1. $\int_0^1 \frac{1-x}{1+x}dx$

2. $\int_0^{\pi/3} \frac{\cos x}{3+4\sin x}dx$

3. $\int_0^{\pi/2} \frac{\sin^2 x}{(1+\cos x)^2}dx$

4. $\int_0^{\pi/2} \frac{\sin x}{\sqrt{1+\cos x}}dx$

5. $\int_0^{\pi} \sin^3 x(1+2\cos x)(1+\cos x)^2 dx$

6. $\int_0^{\infty} \frac{x}{(1+x)(1+x^2)}dx$

7. $\int_0^{\pi/4} \sin 2x \sin 3x\, dx$

8. $\int_0^1 \sqrt{\frac{1-x}{1+x}}dx$

9. $\int_0^{\pi/4} \cos^4 x \sin^3 x\, dx$

10. $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/12}}dx$

11. $\int_0^1 \log(1+x)\, dx$

12. $\int_2^4 \frac{x^2+x}{\sqrt{2x+1}}dx$

13. $\int_0^1 (\cos^{-1} x)^2 dx$

14. $\int_1^2 \frac{x+3}{x(x+2)}dx$

15. $\int_0^{\pi/2} |\sin x - \cos x|\, dx$

16. $\int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right) dx$

17. $\int_{-a}^{a} \frac{x e^{x^2}}{1+x^2} dx$

18. $\int_{0}^{\pi/2} \frac{1}{1+\cot^7 x} dx$

19. $\int_{0}^{\pi/2} \frac{1}{1+\tan^3 x} dx$

20. $\int_{-\pi/4}^{\pi/4} |\tan x| dx$

21. $\int_{0}^{\pi} \frac{x}{1+\cos\alpha \sin x} dx$

22. $\int_{0}^{\pi/2} \frac{\cos^2 x}{\sin x + \cos x} dx$

23. $\int_{0}^{\pi} \cos 2x \log \sin x \, dx$

24. $\int_{0}^{\pi} \frac{x}{a^2 - \cos^2 x} dx, a > 1$

25. $\int_{0}^{\pi} \frac{x \tan x}{\sec x + \tan x} dx,$

26. $\int_{2}^{3} x^{10} \sin^7 x \, dx,$

27. $\int_{0}^{1} \cot^{-1}(1 - x + x^2) \, dx,$

28. $\int_{0}^{\pi/2} \frac{1}{2\cos x + 4\sin x} dx,$

29. $\int_{\pi/6}^{\pi/2} \frac{\operatorname{cosec} x \cot x}{1 + \operatorname{cosec}^2 x} dx,$

30. $\int_{0}^{\pi/2} \frac{dx}{4\cos x + 2\sin x} dx,$

31. $\int_{0}^{2}(2x^2+3)\,dx$

32. $\int_{1}^{4}(x^2+x)\,dx$

33. $\int_{-1}^{1}e^{2x}\,dx$

34. $\int_{2}^{3}e^{-x}\,dx$

35. $\int_{1}^{3}(x^2+3x)\,dx$

36. $\int_{0}^{3}(x^2+1)\,dx$